COHERENCE – The Multidisciplinary Research Journal
Volume No: 4, Issue No: 1, Year: 2013 (January-June)
PP: 42-44
ISSN: 0976-5301
Peer Reviewed & Refereed Journal
Journal Website www.coherencejournal.in

An Investigation into the Role of Brouwer's Fixed Point Theorem in Mathematical Analysis

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Abstract

The main aim of this paper to prove mean value theorem by fixed point theorem .Fixed point is one of the significant factors to the development of mathematics. It should be transformed to the needs of the time and changing scenario of mathematics in the world. In particular, the higher education and the mode of its delivery should be tuned time and again for greater development and changes to cope with such challenges.

Keyword: fixed point, Lipchitz constant ,convex subset.

Introduction

Brouwer's fixed point theorem is an algebraic topology theorem that was proposed and proved by the Dutch mathematician L.E.J. Brouwer in 1912. Brouwer examined the behaviour of continuous functions (continuity) transferring the ball of unit radius in n-dimensional Euclidean space into itself, inspired by earlier work by French mathematician Henri Poincaré. A function is continuous in this context if it maps close points to close points. Brouwer's fixed point theorem states that for any function f, there exists at least one point x where f(x) = x, or where the function f maps x to itself. A fixed point of the function is a point like this. Brouwer's theorem can be reduced to a one-dimensional case and shown to be equivalent to the intermediate value theorem, which states that if a continuous real-valued function f defined on the closed interval [−1, 1] satisfies $f(-1) < 0$ and $f(1) > 0$, then f(x) = 0 for at least one number x between 1 and 1; more formally, an unbroken curve passes through every value between its endpoints Brouwer's fixed point theorem was demonstrated to be equal to an n-dimensional version of the intermediate value theorem in 1940.

Brouwer's theorem does not apply to many other fixed point theorems, including one for the sphere, which is the surface of a solid ball in three-dimensional space. Any continuous function mapping the sphere into itself either has a fixed point or maps some point to its antipodal point, according to the fixed point theorem for the sphere. Fixed point theorems are existence theorems in the sense that they affirm the presence of objects, such as solutions to functional equations, but not necessarily methods for discovering them. Some of these theorems, however, are accompanied with algorithms that generate solutions, particularly for issues in modern applied mathematics.

Theorem . Given a continuous function $f: [a, b] \rightarrow: [a, b], \exists$ a point $c \in [a, b]$ such that $f(c) = c$ while dealing with one dimension , any closed and convex subset of 1R is homomorphic to : [0,1].

$\therefore$ we will show that Brouwer's fixed point Theorem do the case in [0,1].

Proof: Let $f: [0,1] \rightarrow [0,1]$ is continuous function on unit square.

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The function $g(x) = f(x) - x$ is also continuous on unit square as it is difference of two continuous function.

The identity function $f(x) = x$ is also continuous.

When$x = 0, f$
$f(0) \geq 0 \; and \; g(0) = f(0) - 0$

$\Rightarrow g(0)$ is either positive or 0

Now,when$x = 1, f(1) \leq 1 and \; g(1) = f(1) - 1$
$\Rightarrow \; g(1)$ is either negative or 0

Since g is continuous function on a closed set.The inter mediate value theorem applies.

Then we have $g(0) \geq 0 \; and \; g(1) \leq 0$

$\therefore$ there must exist at least one point $c \in [0,1]$ such that $g(c) = 0$

Then, $g(c) = f(c) - c = 0$

$\Rightarrow f(c) = c$

$\Rightarrow f$ has a fixed point .

Brouwer's fixed point Theorem applications

Each of the mapping p_n has a fixed point $x_n \in k_n \subseteq k$. Since k is compact ,the sequence $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ which converges ,say $\lim_{k\to\infty} x_{n_k} = x \in k$

Now observe that for any x

$$\begin{aligned}\left\|f_{n(x)} - f(x)\right\| &= \left\|\sum_{i=1}^{N_n} \alpha_{i(x)}(y_i - f(x) / \sum_{i=1}^{N_n} \alpha_{i(x)}\right\| \\ &\leq [\tfrac{1}{n}\sum_{i=1}^{N_n} \alpha_{i(x)} / \sum_{i=1}^{N_n} \alpha_{i(x)} \\ &= \frac{1}{n}\end{aligned}$$

$\Rightarrow x$ is a fixed point f.

Ex: Consider a metric space (x,d) where x=1R.

Let $f: [a, b] \to [a, b]$ be a differential function equation that has a Lipchitz constant k less than 1.

Then ∃ a unique $x^* \in [a, b]$ such that $f(x^*) = x^*$

Proof: for any $x, y \in [a, b]$ where $x < y$ By mean value theorem ∃ a point c in $[x, y]$

At which $f(y) - f(x) = f'(c)(y - x)$

$$\begin{aligned}\Rightarrow |f(y) - f(x)| &= |f'(c)(y - x)| \\ &= |f'(c)||(y - x)| \\ &\leq k|(y - x)|\end{aligned}$$

For some positive k ≤ 1, which implies that $f$is a contraction mappinig with bounded factor k and 1R is complete. Therefore by banach fixed point theorem , ∃ unique $x^* \in [a, b]$ such that $f(x^*) = x^*$

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